

## A STUDY ON STOCHASTIC METHOD BASED POWER FLOW COMPUTATION

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### ABSTRACT

This paper describes determination of steady state operation of power system, using genetic algorithm. The achievement of many power flow studies is required by most functions performed in power system. Power flow is an electrical engineering known problem, which determines the power system operation point in the steady-state. This paper presents genetic algorithm based power flow computation, which is a stochastic method.

**KEYWORDS:** Power Optimization, Genetic Algorithm, Adaptive Convergence

### INTRODUCTION

Power flow studies [1] are used to ensure that electrical power transfer from the generator to consumer, through grid system, is stable, reliable and economic. The power flow or load flow [2] problem lies in the obtainment of the bus voltages and then in the calculation of the amount of power in the system generation buses as well as the power flow in the system branches. Power bus in the system has 4 variables, where two of them can be controlled and the other two are related to be system conditions. These variables are:  $P$  expresses the values of active power in the bus;  $Q$  expresses the values of reactive power in the bus;  $|V|$  represents the magnitude of the bus voltage; and,  $\delta$  represents the phase angle of the bus voltage. The values of  $P$  and  $Q$  are positive if the active power is injected in the bus i.e. for generation buses, and negative if the active power is taken from the bus i.e. for load buses. The power system buses are classified according to the variables previously known, in three types:

- Type 1 or Type PQ:  $P_i$  and  $Q_i$  are specified and  $|V_i|$  and  $\delta_i$  are calculated – usually, this type represents the load buses of the system.
- Type 2 or Type PV Bus:  $P_i$  and  $|V_i|$  are specified and  $Q_i$  and  $\delta_i$  are calculated – usually, this type is used to represent generation buses.
- Type 3 or Type V $\delta$  (namely “Slack Bus”):  $|V_i|$  and  $\delta_i$  are specified and  $P_i$  and  $Q_i$  are calculated – this type is a representation of the strongest generation bus of the system.

The bus type 1 usually represents the load buses because the values of  $P$  and  $Q$  are known by the load. An example is the power required for a motor to run. The user knows these values and can control it. If he/she puts more motors on a bus, it is quite simple to know the power. However, it is impossible for the user to control the voltage values ( $|V|$  and  $\delta$ ). It also occurs in our houses, we know the power required but we don't have any kind of control about the voltage levels.

The bus type 2 usually represents the generation buses because in a power plant the values of  $P$  and  $|V|$  can be controlled by the operator. If the operator increases the primary source of energy, the value of  $P$  increases and vice-versa. It means, if the operator decreases the primary source of energy, the value of  $P$  decreases. The same occurs with the value of  $|V|$  but in this case the operator changes the excitation system of the generator. However, for this type of bus, the operator cannot have any control over the  $Q$  values.

Finally, the bus type 3 is generally only one in the power flow calculation. Usually, this bus is the strongest power generation bus in the system. This bus gives a reference for the system.

The presented genetic methodology is based on the minimization of the power mismatches in the power system buses. The principle of the proposed algorithm lies in adopting the chromosomes, as the power system bus voltages, phase angles and other magnitudes. The computational routine starts with estimated initial values for the chromosome parameters, and these values are updated in each iteration process through the genetic operators, and the rule function, which comprises the problem modeling.

## MATHEMATICAL MODEL OF POWER OPTIMIZATION

### Objective Function for Power Optimization

Construction of genetic algorithm for power optimization uses following objective function: [3]

$$F = \Delta S_k = \sqrt{(\Delta P_k)^2 + (\Delta Q_k)^2}$$

Where  $k$  is system bus index,  $\Delta P$  and  $\Delta Q$  are active and reactive power mismatch at bus  $k$  and  $\Delta S_k$  represent the apparent power mismatch at bus  $k$ .

### Constraints

The result obtained from mathematical solution represents a group of solutions. However, this group of solution of system should reflect the practical engineering values so that power system operation must meet certain technical and economic requirement. These requirements impose constraints on some variable which are:

#### Node Bus Voltage

$$V_{i \min} \leq V \leq V_{i \max}$$

#### Active Power and Reactive Power at Node

$$P_{i \min} \leq P \leq P_{i \max}$$

#### Phase Difference between Voltages of Some Node

$$|\delta_i - \delta_j| < |\delta_i - \delta_j|$$

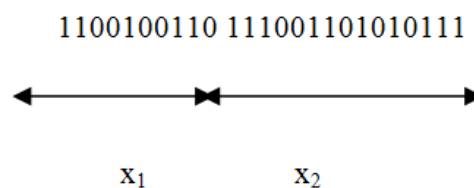
### General Analysis of Genetic Algorithm: [4], [5]

The GA is a stochastic global search method that mimics the metaphor of natural biological evolution. GA operates on a population of potential solutions, applying the principle of 'survival of the fittest' to produce (hopefully) better and better approximations to a solution. At each generation, a new set of approximations is created by the process of

selecting individuals, according to their level of fitness, in the problem domain and breeding them together using (crossover mutation) operators borrowed from natural genetics. This process leads to the evolution of populations of individuals that are better suited to their environment than the individuals that they were created from, just as in natural adaptation.

### a) Initial Population Generation

Individuals, or current approximations, are encoded as strings, *chromosomes*, composed over some alphabet(s), so that the *genotypes* (chromosome values) are uniquely mapped onto the decision variable (*phenotypic*) domain. The most commonly used representation in GAs is the binary alphabet {0, 1} although other representations can be used, e.g. ternary, integer, real-valued etc. For example, a problem with two variables,  $x_1$  and  $x_2$ , may be mapped onto the chromosome structure as:



A binary string and a group of one or more binary bit which represent the value for fitness function is called gene. The chromosome is made up of one or more gene and precision value will define its length. For example a problem requires 1 bit (an integer value of 0 to 15) and problem being solved require 3 genes then chromosome will have length of 12 bits.

Mapping the genes to its values or allele requires the range and the precision of values to be known. For example: in power system, the range of values lies between 0 to 1 in per unit system, so that the range is between 0 to 1 and if the precision required is 0.001, then  $1 \times 1000 = 1000$  equal sizes are needed. This will require 12 bits as  $2^{11} = 2048 \leq 1000 \leq 2^{12} = 4096$ .

### b) Determination of Population Size

The population size affects the outcome of genetic optimization and efficiency. When population size is too small, the genetic algorithm to optimize performance is generally not very good, easy to fall into local optimal solution; and when population size is too large then there is computational complexity.

### c) Fitness Function [6]

There exists a utility function (a user's preference function i.e. objective function) that assigns a utility value (the gained value from the user's perspective) for each item. These values vary from one chromosome to the other (phenotype value). In general, the utility value of a particular chromosome decreases with its ranking order. More formally, given an objective function  $f(r)$ , and two ranks  $r_1, r_2$ , with  $r_1 < r_2$ , according to this assumption, we expect the following condition to hold true:

$$f(r_1) > f(r_2)$$

For example, suppose the top two chromosomes in a search rank list are both relevant. According to Condition (1), the first relevant chromosome in the rank list should give a reader, more utility value than the second one. The further down the rank list, the less utility a relevant chromosome gives. One of the possible functions to satisfy the order preserving condition is given as:

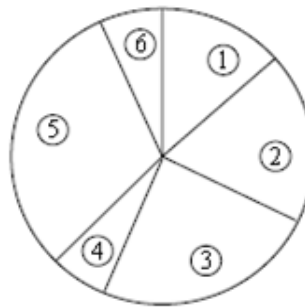
$$f(r) = 1/r$$

Similarly, the fitness functions for given objective function  $F(x)$  can be expressed as:

$$\text{Fit}(f(x)) = 1/f(x)$$

#### d) Selection Method [7]

Selection techniques employ a “roulette wheel” mechanism to probabilistically select individuals based on some measures of their performance. Areal-valued interval, *Sum*, is determined as either the sum of the individual’s expected selection probabilities or the sum of the raw fitness values over all the individuals, in the current population. Individuals are then mapped one-to-one into contiguous intervals in the range  $[0, \text{Sum}]$ . The size of each individual interval corresponds to the fitness value of the associated individual. For example, in Fig. 1 the circumference of the roulette wheel is the sum of all six individual’s fitness values. Individual 5 is the fittest individual and occupies the largest interval, whereas, individuals 6 and 4 are the least fit and have, correspondingly, smaller intervals within the roulette wheel.



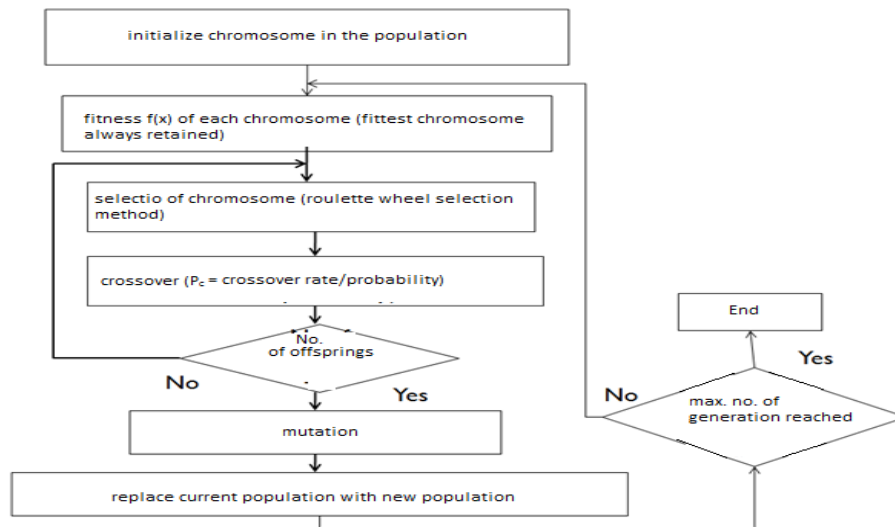
**Figure 1: Roulette Selection**

$$\text{Sum} = \sum_{i=1}^n \text{fitness}[i]$$

$$\text{Selection probability}[i] = \text{fitness}[i] / \text{Sum}$$

- a) **Crossover:** It will merge two parent chromosomes to form a new child chromosome. The schema in each child will come from both parents. The crossover operation ideally takes shortest length, high fitness schema of both parents and mates them to form two children with short length, high fitness schema remaining intact in the children. Assuming this happen, the child will have higher fitness value than either of the parents. In this paper uniform crossover is taken for study purpose.
- b) **Mutation:** Mutation is used to introduce new allele to the population. The mutation operator works by randomly selecting a candidate from the population and then randomly selecting an allele to change. In binary mutation, the chromosome and bit position will be selected at random. The binary mutation involves flipping of a bit in chromosome.

After mutation and crossover, new genetic population is created and same process is repeated.



**Figure 2: GA Algorithm**

#### ALGORITHM FOR COMPUTATION: [8]

- a) The system bus voltages are assigned with chromosome Value which is given as:

$$X_i = \{\delta_2, \delta_3, \dots, \delta_k, \dots, \delta_n, |V_2|, |V_3|, \dots, |V_k|, \dots, |V_n|\}$$

- b) The reactive power of PV buses, active and reactive power of V0 bus are computed by applying equation as below:

$$P_i - jQ_i - y_{i1}V_1V_i^* - \dots - y_{in}V_nV_i^* = 0$$

- c) The power flow in a system branch is calculated using equation:

$$S_{ij} = P_{ij} + jQ_{ij} = V_i(V_i^* - V_j^*)y_{ij} + V_iV_i^*y_{sh,i}$$

Where S represent the complex apparent power between the buses i and j,  $P_{ij}$  is active power between the buses i and j,  $Q_{ij}$  is the reactive power between the buses i and j,  $V_i$  is the ith bus voltage,  $V_j$  is the jth bus voltage,  $y_{ij}$  is the admittance between buses i and j and  $y_{sh}$  is the shunt admittance of the bus.

- d) The active and reactive power mismatch of each bus is calculated as the sum of injected power in approached bus using objective function equations discussed above.
- e) The performance index is computed for each chromosome by fitness function. Then probability of selection of each chromosome is calculated by roulette wheel selection criteria.
- f) The probability of selection is multiplied by number of individual chromosome, resulting in final degree of each chromosome.
- g) The chromosome which has the worst (the biggest) power mismatch until now is kept and is used in mutation.

Once all chromosomes have passed through described routine, it then further proceeds as follows:

- h) The mating pool for next generation is composed, according to degree of each chromosome. For example, if degree of each chromosome is 4.27, this chromosome has 4 copies in mating pool.
- i) Crossover operator with crossover probability  $P_c$  and mutation operator with mutation probability  $P_m$  is applied to directly affect the convergence of algorithm.  $P_c$  and  $P_m$  can be applied through function as given below: [9]

$$P_c = k_1, \quad f' < f_{avg}$$

$$k_2 (f_{max} - f') / (f_{max} - f_{avg}), \quad f' \geq f_{avg}$$

$$P_m = k_3, \quad f' < f_{avg}$$

$$k_4 (f_{max} - f') / (f_{max} - f_{avg}), \quad f' \geq f_{avg}$$

Where:  $f_{max}$  is maximum fitness value of group;

$f_{avg}$  is the average fitness;  $f$  is the individual fitness value

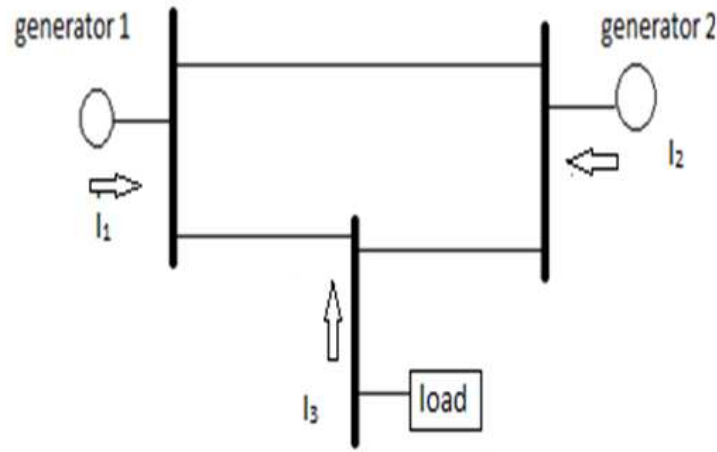
$k_1$  and  $k_3$ : represent crossover rate and mutation rate respectively, when the individual's fitness value is less than the average population fitness value.

$k_2$  and  $k_4$  represent constants maximum cross-rate and mutation rate respectively, when an individual's fitness value is greater than the average population fitness value.

Above equation involves adaptive convergence technique for further generations.

### EXAMPLE ANALYSIS

A simple 3 bus system is represented by single line diagram as shown in figure 3.



**Figure 3: Bus 1 is a V- $\delta$  bus, Bus 2 is a PV Bus, and Bus 3 is a PQ Bus**

Admittance matrix is assumed to be known such that  $Y_{12} = -4j$ ,  $Y_{13} = -5j$ ,  $Y_{23} = -3j$  is given and shunt admittance is assumed to be zero and  $|V_1| = 1$ ,  $|V_2| = 1.1$ ,  $\delta_1 = 0.00$

### Error Optimizing Function (Objective Function)

$$E = \sqrt{(P_2 - P_{2c})^2 + (P_3 - P_{3c})^2 + (Q_3 - Q_{3c})^2}$$

P2c, P3c, Q3c represents the calculated values and P2=1.0, P3=0.5, Q3= -0.2 are the given values corresponding to specific bus. The GA parameter used in this paper is:

- a) Size of population = 10
- b) Fitness scaling = proportional
- c) Crossover function = Heuristic
- d) Mutation function = Gaussian
- e) Constraints =  $0.9 \leq V \leq 1$

$$-0.25 \leq \delta \leq 0.25$$

Calculation of first iteration for initial population is given in table 1 and second generation population is given in table 2.

**Table 1: Calculation for First Iteration**

|    | 1 <sup>st</sup><br>Generation | 1  | 2                       | 3                   | 4                   | 5                   | 6                    | 7            | 8                   | 9                    | 10              |
|----|-------------------------------|--|-------------------------|---------------------|---------------------|---------------------|----------------------|--------------|---------------------|----------------------|-----------------|
| 1  | $\delta_2$ (radians)          | 0  | 0.17<br>5               | 0.175               | 0.175               | 0.175               | -0.175               | -0.175       | -<br>0.175          | -0.175               | -0.175          |
| 2  | $ V_3 $                       | 1  | 1.1                     | 1.1                 | 0.9                 | 0.9                 | 1.1                  | 0.9          | 0.9                 | 0.9                  | 0.937           |
| 3  | $\delta_3$ (radian)           | 0  | 0.17<br>5               | -<br>0.175          | -<br>0.175          | 0.175               | -0.175               | -0.175       | 0.175               | -0.175               | 0.106           |
| 4  | $I_1$                         | 0.0+0.4j   | -<br>1.91<br>+1.8<br>3j | 0.19+<br>0.75j      | -1.54-<br>.24j      | 0.017<br>-0.23j     | -<br>0.19+0<br>.748j | 1.72+0.75j   | -<br>.0174<br>-.23j | 1.54-<br>.24j        | -0.926          |
| 5  | $I_2$                         | 0.0-0.7j   | 0.76<br>-<br>0.33j      | 1.91-<br>0.33j      | 0.87-<br>0.92j      | 1.81-<br>0.92j      | -1.91-<br>0.33j      | -0.76-0.33j  | -1.8-<br>0.923<br>j | -0.87-<br>0.92j      | 0.52-<br>0.86j  |
| 6  | $I_3$                         | 0.0+0.3j   | 0.95<br>-<br>0.41<br>5j | -2.1-<br>0.415<br>j | 0.68+<br>10.16<br>j | 0.67+<br>10.16<br>j | 2.10-<br>0.416j      | -0.95-0.416j | 1.82+<br>10.16<br>j | -<br>0.67+<br>1.159j | 0.44+0.<br>827j |
| 7  | $P_2$                         | 0  | 0.88<br>6               | 2.132               | 1.11                | 2.13                | -2.14                | -0.886       | -2.12               | -1.12                | 0.57            |
| 8  | $P_3$                         | 0  | 0.95<br>4               | -2.35               | 0.42                | 0.775               | 2.201                | -1.11        | 1.43                | -0.34                | 0.32            |
| 9  | $Q_3$                         | 0.3j   | 0.63<br>3               | -<br>0.047          | 1.13                | 0.923               | 0.85                 | -0.267       | 1.3                 | 1.51                 | 0.81            |
| 10 | E                             | 1.15   | 0.95<br>2               | 3.07                | 1.33                | 1.61                | 3.7                  | 2.4          | 3.58                | 2.85                 | 1.112           |
|    | f (x)                         | 0.86   | 1.05                    | 0.32                | 0.75                | 0.62                | 0.27                 | 0.41         | 0.28                | 0.35                 | 0.89            |
|    | sum                           | 0.869+1.05+0.32+0.75+0.62+0.27+0.41+0.28+0.35+0.89=5.809 |                         |                     |                     |                     |                      |              |                     |                      |                 |
|    | $f_{avg}(x)$                  | 5.809/10 = 0.589= 0.6                                    |                         |                     |                     |                     |                      |              |                     |                      |                 |
|    | $f_{max}(x)$                  | 1.5 (assumed)  |                         |                     |                     |                     |                      |              |                     |                      |                 |
|    | Selection<br>Probability      | 0.14   | 0.18                    | 0.055               | 0.13                |                     | 0.1                  | 0.046        | 0.07                | 0.048                | 0.15            |
|    | No. of Copies                 | 1  | 2                       | 0                   | 1                   |                     | 1                    | 0            | 0                   | 0                    | 1               |
|    | Rank                          | 3  | 1                       | 6                   | 4                   |                     | 5                    | 9            | 7                   | 10                   | 2               |

Here,  $P_c = 0.8$ , for  $f < f_{avg}$

$$0.7 \times [(f_{max} - f) / (f_{max} - f_{avg})], \text{ for } f > f_{avg}$$

$P_m = 0.9$ , for  $f < f_{av}$

$$0.8 \times [(f_{max} - f) / (f_{max} - f_{avg})], \text{ for } f > f_{avg}$$

**Table 2: Second Generation Population**

| S. No | 2 <sup>nd</sup> Generation | 1      | 2      | 3    | 4      | 5     | 6               | 7               | 8               | 9               | 10              |
|-------|----------------------------|--------|--------|------|--------|-------|-----------------|-----------------|-----------------|-----------------|-----------------|
|       |                            | Best   | Best   | Best | Best   | Best  | Breed from Best | Breed from Best | Breed from Best | Breed from Best | Breed from Best |
| 1     | $\delta_2$ (radian)        | -0.175 | 0.107  | 0    | 0.175  | 0.107 | -0.175          | -0.14           | 0.101           | 0               | 0.203           |
| 2     | $V_3$                      | 1.1    | 0.937  | 1    | 0.9    | 0.937 | 1.1             | 0.88            | 0.88            | 0.95            | 1.04            |
| 3     | $\delta_3$ (radian)        | -0.175 | -0.106 | 0    | -0.175 | 0.106 | -0.175          | -0.14           | -0.1            | 0               | -0.203          |

Algorithm mentioned is iterated number of times, to get the required accuracy of the result.

## RESULTS

Optimal value of variable above determines the steady state operation of power system.

| Control Variable | $\delta_2$ (radian) | $V_3$ (volt) | $\delta_3$ (radian) |
|------------------|---------------------|--------------|---------------------|
| Optimal Value    | -0.177              | 1.049        | -0.23               |

The optimal value of control variables indicates the minimum power loss in the power system. The result obtained below gives the optimal value of variable that determines the steady state operation corresponding to minimum value of objective function which actually indicates the minimum power loss in the system.

## CONCLUSIONS

An algorithm has been developed to determine the steady state operating point of power system. It is based on the concept of genetic algorithms. The methods of constraint satisfaction are developed in the paper for satisfying the specified powers of the PQ nodes and the specified voltage magnitudes of the PV nodes. Three mechanisms, the dynamic population technique, the solution convergence technique and the selection method for nodal voltage updating, have been developed and incorporated to enhance its performance and computational speed.

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